Logics of Hyperintensional Practical Reasons

It is widely assumed in contemporary normative theory that normative reasons are a key element in explaining or grounding other normative concepts, or that reasons themselves are the fundamental normative notions. In this paper we argue that normative reasons are hyperintensional, and put forward a formal account of this thesis. That reasons are hyperintensional means that logically equivalent propositions may be different reasons for the same action (duty, ought, etc.), and therefore cannot be substituted for each other. Be they a fundamental or derived concept, as far as we know so far there has been no explicit formal proposal to deal with normative reasons precisely, with the exception of [4]. We propose to remedy this fact by introducing a hyperintensional logic for normative reasons based on justification logic [1].

We make three simplifying assumptions: *first*:

> A normative reason for agent N to ϕ is an explanation of why N ought to ϕ [2, p. 50].

Second, we consider only *pro toto* (normative) reasons,
Third, we work within a somewhat static account, ignoring for instance enablers and disablers, intensifiers and attenuators.

Three arguments for the hyperintensionality of reasons

1. Logically equivalent normative reasons are not normatively substitutable. Consider A and $A \vee (A \wedge B)$, which are logically equivalent, e.g. $A =$ 'you have a license', $B =$ 'you have a license or you have a license and you are drunk'. As reasons for driving they are clearly not normatively equivalent, though they are logically equivalent. If reasons were not hyperintensional these would be equally good reasons to drive.

2. Hyperintensionality has greater explanatory power. If normative reasons are hyperintensional, there is a principled, independently motivated reason to reject *Reasons Inheritance*:

> If ϕ implies ψ, then if x is a normative *pro toto* reason for ϕ, then x is a normative *pro toto* reason for ψ.

This enables a response to puzzles such as Ross's, the Good Samaritan, and Free Choice Permission.

3. Normative reasons appear to have two components, a non-causal explanation and an ought, see e.g. [2]. There is good consensus that non-casual explanations are hyperintentional, and hence if normative reasons are non-causal explanations that is strong *prima facie* evidence that normative reasons are too.

Towards a hyperintensional logic for reasons

We are interested not only in *what* we ought to do, but also in *how* certain considerations support what we ought to do. The explicit modal language of Justification Logic [1] offers us a formal framework for such an approach.

In the explicit language formulas of the form $\Box A$ are replaced with formulas of the form $t{:}A$, where $t{:}$ is called a *term*, the deontic reading of which is

$$t \text{ is a reason to do } A$$

or

$$t \text{ is a reason why } A \text{ is obligatory.}$$

$t{:}(A \to B)$ expresses that t is a reason to do B, on the condition that A holds.

In addition to terms denoting specific reasons the explicit modal language can also contain operations on terms. For instance, say that s *is a reason to do A* and t *is a reason to do B given that A*. By applying s to t we obtain a complex reason to do B, denoted by the complex term $t{\cdot}s$. This is expressed by the principle $t{:}(A \to B) \to (s{:}A \to (t{\cdot}s){:}B)$.

Terms may be either variables, $x, y, z \ldots$ or constants $c_1, c_2 \ldots$

Definition 1 (Normative Reasons Logic NRL)**.**

A0. Axioms of classical propositional logic.

A1. $t{:}(A \to B) \to (s{:}A \to (t \cdot s){:}B)$

R1. Modus Ponens

R2. Axiom Necessitation: $\dfrac{\vdash A}{\vdash c{:}A}$ where A is any of A0 or A1 and c is some constant.

This is a very weak logic, weaker even than the basic system of alethic justifications, J [1].[1] We consider also some stronger systems adding further constraints on normative reasons.

Following standard deontic logic we consider adding the explicit version of the D principle to NRL:

$$\mathsf{NRLD} = \mathsf{NRL} + t{:}A \to \neg t{:}\neg A.$$

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[1] J contains the $+$ operation, which expresses a kind of monotonicity, given by the axiom scheme $s{:}A \to (s + t){:}A$ where $(s + t)$ is the result of adding further reason, t, to s, for instance by adding more lines to a proof of A. Such monotonicity does not seem appropriate for normative reasons, cf. the reasons for driving above.

We can consider this the basic system of *pro toto* normative reasons. Of course were we to consider *pro tanto* reasons, such principle would likely be dropped.

We consider that normative reasons should be justifiable in terms of our background normative theory. This can be expressed by the principle $t{:}A \to\ !t{:}t{:}A$, *if A is obligatory for reason t then !t is a justification for the claim that A is obligatory for reason t*:

$$\mathsf{NRL4} = \mathsf{NRL} + t{:}A \to !t{:}t{:}A.$$

While clearly $t{:}A \to A$ does not hold for normative reasons, a weaker version, $s{:}t{:}A \to t{:}A$, can be considered, which claims that justifications for normative reasons are factive. With this addition we get the logic:

$$\mathsf{NRLWR} = \mathsf{NRL} + s{:}t{:}A \to t{:}A.$$

Philosophical Import

Is our logic truly hyperintensional? It is; suppose that A and B are both obligatory, and hence both true in all (accessible) deontically possible worlds and hence in that sense deontically equivalent: from this one cannot conclude that if t is a reason why A is obligatory that t is also why B is obligatory.[2]

This also enables us to avoid the deontic puzzles which depend on Resaons Inheritance. Given that $A \to B$ and that $t{:}A$ it does not follow that $t{:}B$. Consider Ross's Puzzle in the explicit language. Since $\vdash A \to (A \vee B)$ is an axiom, by axiom necessitation we derive $\vdash c{:}(A \to (A \vee B))$. Now assume that $t{:}A$ (t is a reason why mailing the letter is obligatory). By axiom A1 and modus ponens we can derive $(c{\cdot}t){:}(A \vee B)$, which is clearly not the same obligation. We may well have the obligation to either mail the letter or burn it, given we have an an obligation to mail it *and also* a reason why 'it is obligatory to either mail it or burn it on condition we mail it'. The obligation to mail or burn is a different, derived, obligation, which does not depend solely on the obligation to mail the letter.

References

[1] Sergei Artemov. "The Logic of Justification". In: *Review of Symbolic Logic* (2008) (cit. on pp. 1, 2).

[2] John Broome. *Rationality Through Reasoning*. Oxford University Press, 2013 (cit. on pp. 1, 2).

[3] Melvin Fitting. "The Logic of Proofs, Semantically". In: *Annals of Pure and Applied Logic* 132 (2005), pp. 1–25 (cit. on p. 3).

[4] John F. Horty. *Reasons as Defaults*. Oxford University Press, 2012 (cit. on p. 1).

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[2]This should be clear informally and intuitively, but the point can be made formally in a suitable Fitting model, see [3].